Response Statistics and Failure Probability Determination of Nonlinear Stochastic Structural Dynamical Systems

Novel approximation techniques are proposed for the analysis and evaluation of nonlinear dynamical systems in the field of stochastic dynamics. Efficient determination of response statistics and reliability estimates for nonlinear systems remains challenging, especially those with singular matrices or endowed with fractional derivative elements. This thesis addresses the challenges of three main topics.

The first topic relates to the determination of response statistics of multi-degree-of-freedom nonlinear systems with singular matrices subject to combined deterministic and stochastic excitations. Notably, singular matrices can appear in the governing equations of motion of engineering systems for various reasons, such as due to a redundant coordinates modeling or due to modeling with additional constraint equations. Moreover, it is common for nonlinear systems to experience both stochastic and deterministic excitations simultaneously.

In this context, first, a novel solution framework is developed for determining the response of such systems subject to combined deterministic and stochastic excitation of the stationary kind. This is achieved by using the harmonic balance method and the generalized statistical linearization method. An over-determined system of equations is generated and solved by resorting to generalized matrix inverse theory.

Subsequently, the developed framework is appropriately extended to systems subject to a mixture of deterministic and stochastic excitations of the non-stationary kind. The generalized statistical linearization method is used to handle the nonlinear subsystem subject to non-stationary stochastic excitation, which, in conjunction with a state space formulation, forms a matrix differential equation governing the stochastic response. Then, the developed equations are solved by numerical methods.

The accuracy for the proposed techniques has been demonstrated by considering nonlinear structural systems with redundant coordinates modeling, as well as a piezoelectric vibration energy harvesting device have been employed in the relevant application part.

The second topic relates to code-compliant stochastic dynamic analysis of nonlinear structural systems with fractional derivative elements. A novel approximation method is first proposed to efficiently determine the peak response of nonlinear structural systems with fractional derivative elements subject to excitation compatible with a given seismic design spectrum. The proposed methods involve deriving an excitation evolutionary power spectrum that matches the design spectrum in a stochastic sense. The peak response is approximated by utilizing equivalent linear elements, in conjunction with code-compliant design spectra, hopefully rendering it favorable to engineers of practice. Nonlinear structural systems endowed with fractional derivative terms in the governing equations of motion have been considered. A particular attribute pertains to utilizing localized time-dependent equivalent linear elements, which is superior to classical approaches utilizing standard time-invariant statistical linearization method.

Then, the approximation method is extended to perform stochastic incremental dynamical analysis for nonlinear structural systems with fractional derivative elements exposed to stochastic excitations aligned with contemporary aseismic codes. The proposed method is achieved by resorting to the combination of stochastic averaging and statistical linearization methods, resulting in an
efficient and comprehensive way to obtain the response displacement probability density function. A stochastic incremental dynamical analysis surface is generated instead of the traditional curves, leading to a reliable higher order statistics of the system response.

Lastly, the problem of the first excursion probability of nonlinear dynamic systems subject to imprecisely defined stochastic Gaussian loads is considered. This involves solving a nested double-loop problem, generally intractable without resorting to surrogate modeling schemes. To overcome these challenges, this thesis first proposes a generalized operator norm framework based on statistical linearization method. Its efficiency is achieved by breaking the double loop and determining the values of the epistemic uncertain parameters that produce bounds on the probability of failure a priori. The proposed framework can significantly reduce the computational burden and provide a reliable estimate of the probability of failure.